Capacity Analysis of Runway Systems

Dr. Raik Stolletz
Department of Production Management
Hannover School of Economics and Business Administration
Leibniz University of Hannover

Outline

1. Characterization of Capacity and Demand
2. Overview of Performance Models
3. Stationary Backlog-Carryover (SBC) Approach
   (a) Main Idea
   (b) Approximation of the Utilization
   (c) Modified Arrival Rate (MAR) Approximation
4. Numerical Results
5. Conclusion

Characterization of Capacity and Demand (1)

- Capacity:
  - layout of the runway system,
  - operation and aircraft mix (separation requirement),
  - weather conditions,
  - stochastic variation in processing times
- Demand:
  - stochastic (delayed arriving aircraft, delayed passengers at the gate)
  - highly dynamic

Characterization of Capacity and Demand (2)

Airport queues are

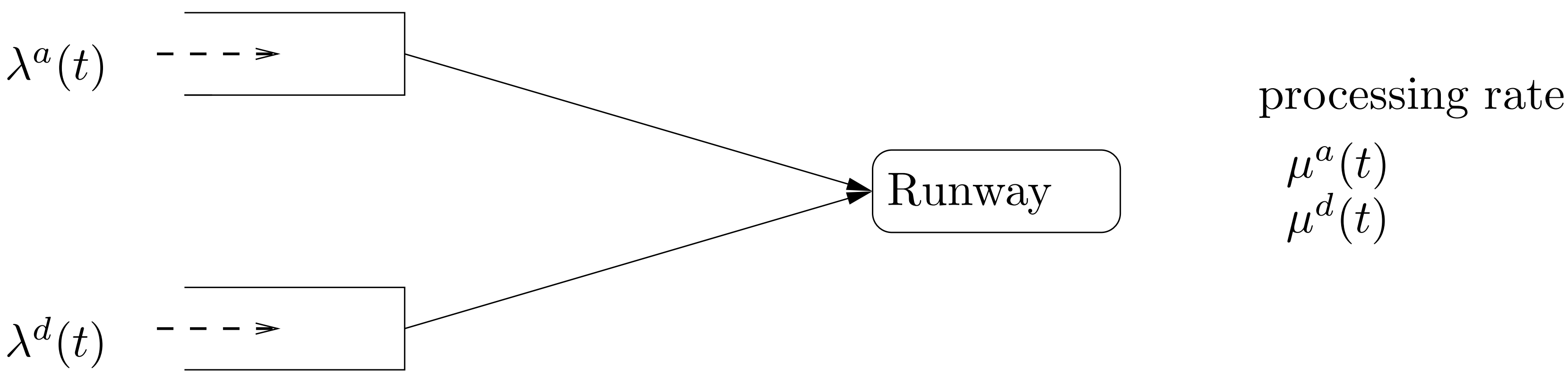
- strongly non-stationary,
- can be modelled as stochastic processes, and
- the demand rate may exceed the capacity for an extended period of time (temporal overloading).

Performance analysis and optimization needs **stochastic and non-stationary queueing systems** with the possibility of **overloading**.

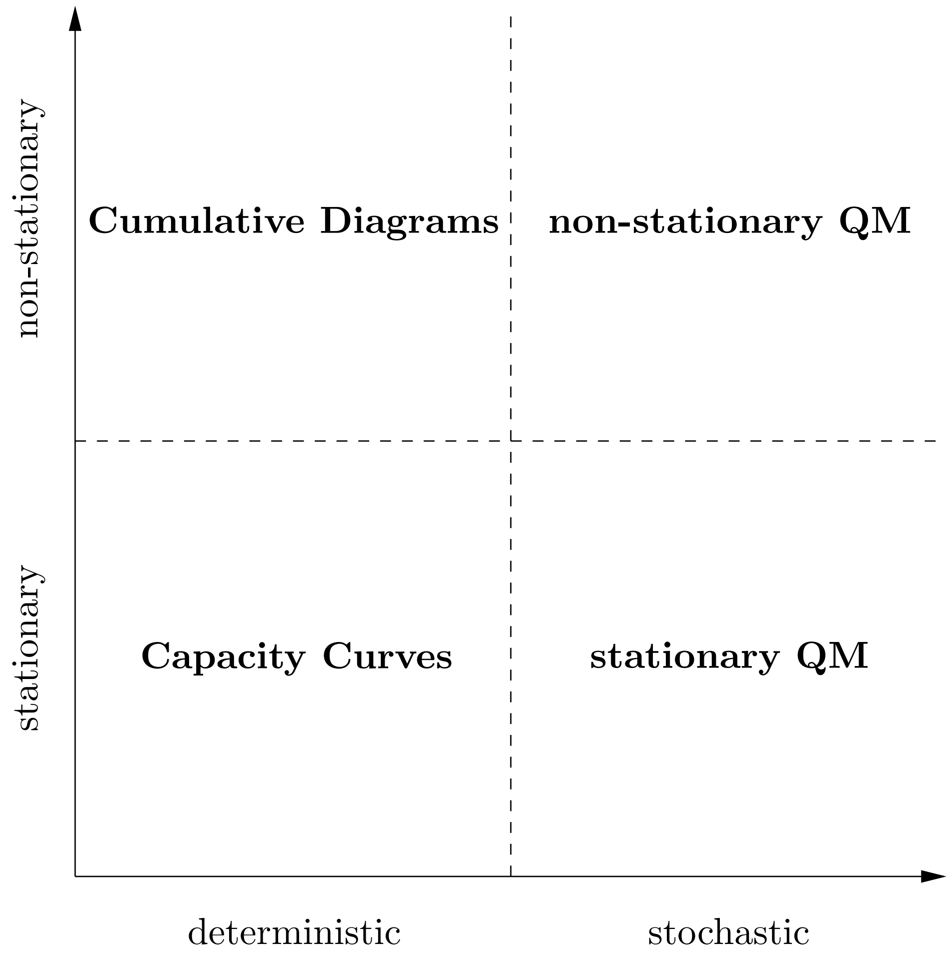
A simple Runway Model

Landings and takeoffs share a common runway:

$M/H_2/1$ queueing model



Overview of Performance Models (1)



Overview of Performance Models (2)

Approximation of Non-stationary Queueing Models

- Cumulative Diagrams/Fluid Approximation (Mandelbaum and Massey (1995), Hansen (2002))
- approximation with stationary queueing models (*SSA* (Green et al. (1991)), *PSA* (Green and Kolesar (1991)), *SIPP* (Green et al. (2001)))
- others (Ingolfsson et al. (2003), *randomization* (Grassmann (1977)), *infinite server* (Jennings et al. (1996), Massey and Whitt (1997)))

SBC-Approach: Main Idea (1)

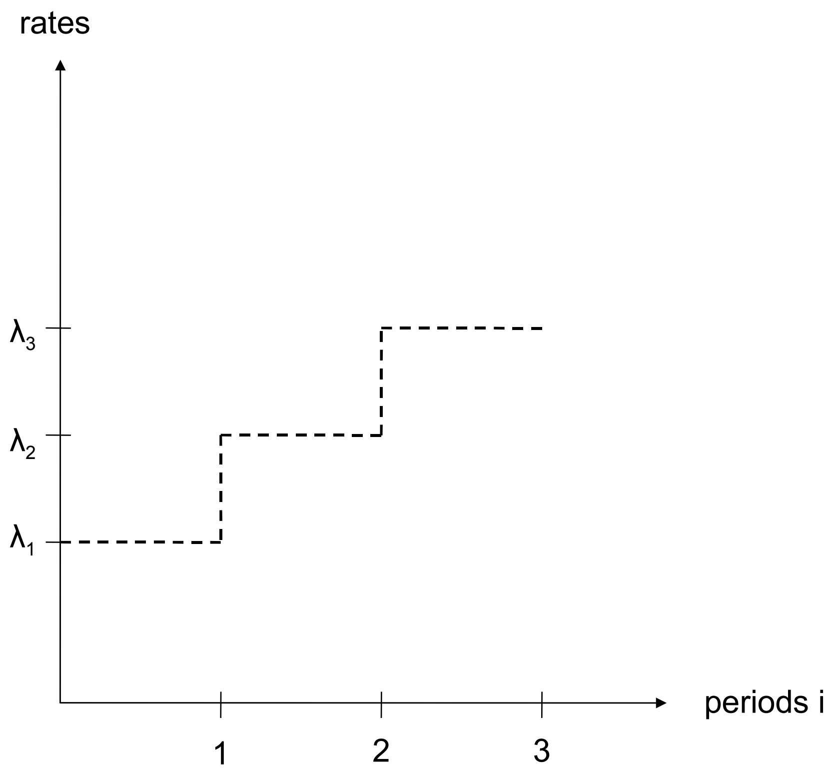
1. Division of the time interval into small periods (constant rates)

2. Approximation of the utilization:

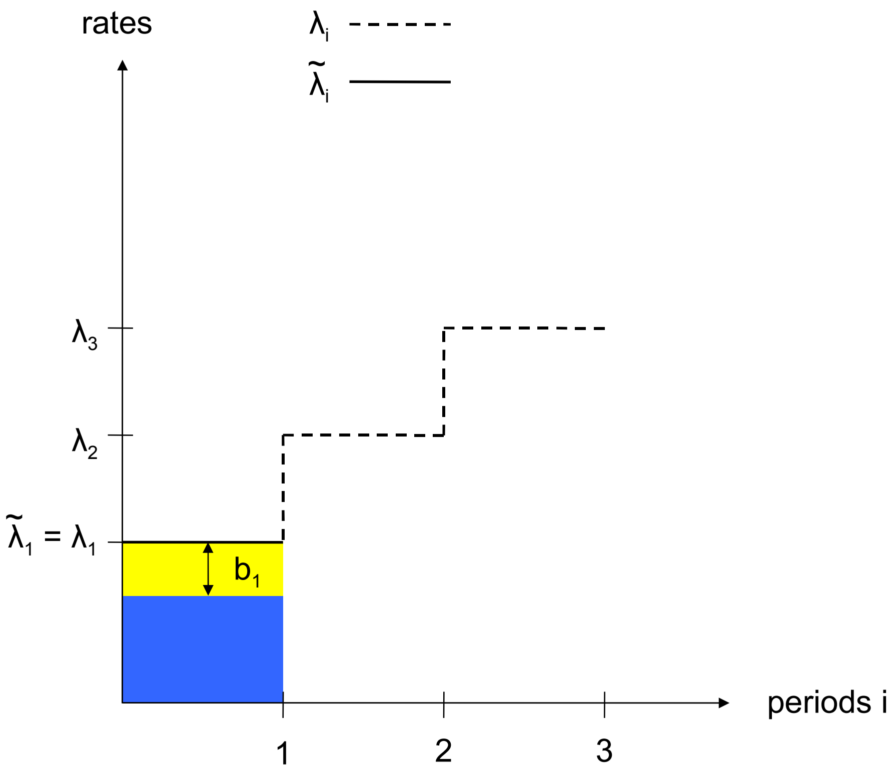
   (a) stationary loss model
   (b) backlog generation from artificial blocking
   (c) carryover as additional arrivals in future periods

3. Approximation of waiting-based performance measures with a stationary waiting model
   (*Modified Arrival Rate (MAR)-approximation*)

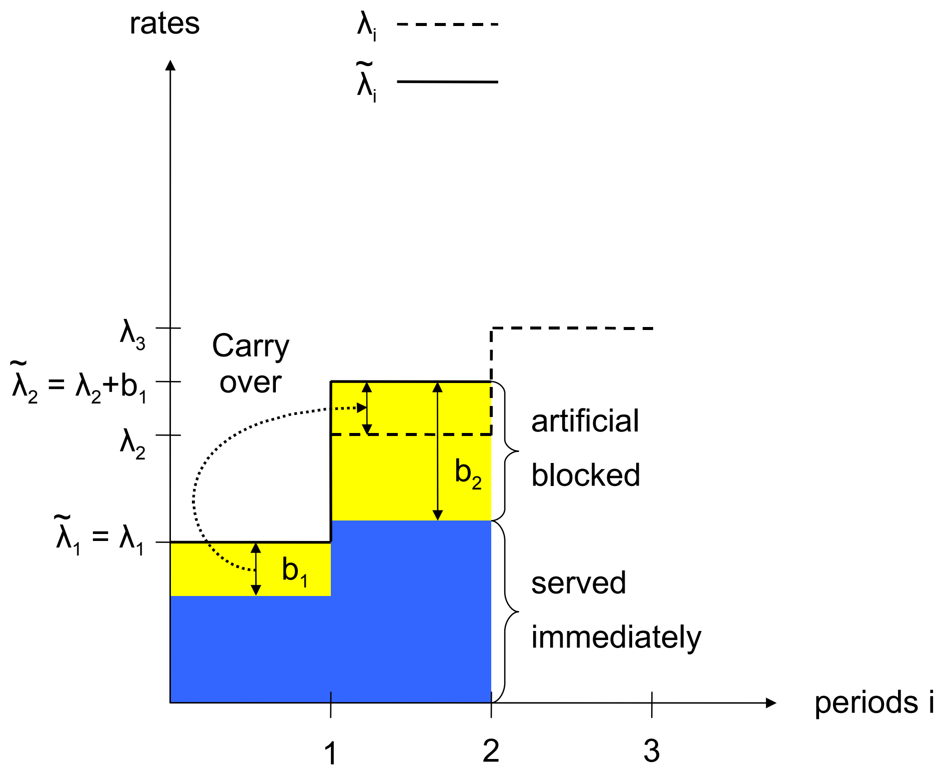
SBC-Approach: Main Idea (2)



SBC-Approach: Main Idea (2)



SBC-Approach: Main Idea (2)



SBC-Approach: Main Idea (3)

SBC-Approximation of expected utilization: Define *backlog rate* b_i and *artificial arrival rates* $\widetilde{\lambda}_i$

1. Initialization: $\widetilde{\lambda}_1 = \lambda_1$ and $b_0 = 0$

2. For $i = 1$ to T:

   (a) Solve appropriate loss system with $\widetilde{\lambda}_i$ and μ_i

   (b) Backlog generation: $b_i = \widetilde{\lambda}_i \cdot P_i(B)$

   (c) Backlog carryover: $\widetilde{\lambda}_{i+1} = \lambda_{i+1} + b_i$

3. Results: $E[U_i]$, $P_i(B)$, b_i

SBC-Approach: Approximation of the Utilization

SBC-approximation of the expected utilization:

- artificial arrival rates:

$$\widetilde{\lambda}_i^a = \lambda_i^a + b_{i-1}^a \quad \text{and} \quad \widetilde{\lambda}_i^d = \lambda_i^d + b_{i-1}^d$$

$$\widetilde{\lambda}_i = \widetilde{\lambda}_i^a + \widetilde{\lambda}_i^d$$

- average processing rate

$$\mu_i = \left(\frac{\widetilde{\lambda}_i^a}{\widetilde{\lambda}_i \mu_i^a} + \frac{\widetilde{\lambda}_i^d}{\widetilde{\lambda}_i \mu_i^d}\right)^{-1}$$

- common probability of blocking (Erlang loss system)

$$P_i(B) \quad = \quad \frac{\widetilde{\lambda}_i/\mu_i}{1+\widetilde{\lambda}_i/\mu_i}.$$

- backlog rates

$$b_i^a = \widetilde{\lambda}_i^a \cdot \frac{\widetilde{\lambda}_i/\mu_i}{1+\widetilde{\lambda}_i/\mu_i} \quad \text{and} \quad b_i^d = \widetilde{\lambda}_i^d \cdot \frac{\widetilde{\lambda}_i/\mu_i}{1+\widetilde{\lambda}_i/\mu_i}.$$

- expected utilization

$$E[U_i] \quad = \quad E[U_i^a] + E[U_i^d] = \frac{\widetilde{\lambda}_i^a(1-P_i(B))}{\mu_i^a} + \frac{\widetilde{\lambda}_i^d(1-P_i(B))}{\mu_i^d}.$$

SBC-Approach: Approximation of the Queue Length (1)

1. Approximation of the utilization: stationary loss model

2. Approximation of waiting-based performance measures: stationary waiting model
   Modified Arrival Rate (MAR)-approximation:

   (a) Chose modified arrival rate λ_i^{MAR} such that the SBC-utilization $E[U_i]$ is reached.
   (b) Compute $E[L_i]$, $E[N_i]$, and $E[W_i]$ for the waiting system with this modified arrival rate λ_i^{MAR}.

SBC-Approach: Approximation of the Queue Length (2)

Stationary waiting system: $M/H_2/1/\infty$

1. Determination of modified arrival rates:

$$\lambda_i^{a,MAR} = E[U_i^a]\,\mu_i^a \quad \text{and} \quad \lambda_i^{d,MAR} = E[U_i^d]\,\mu_i^d,$$

2. Determination of performance measures:
   Pollaczek-Khintchine formula for the $M/G/1$ model:

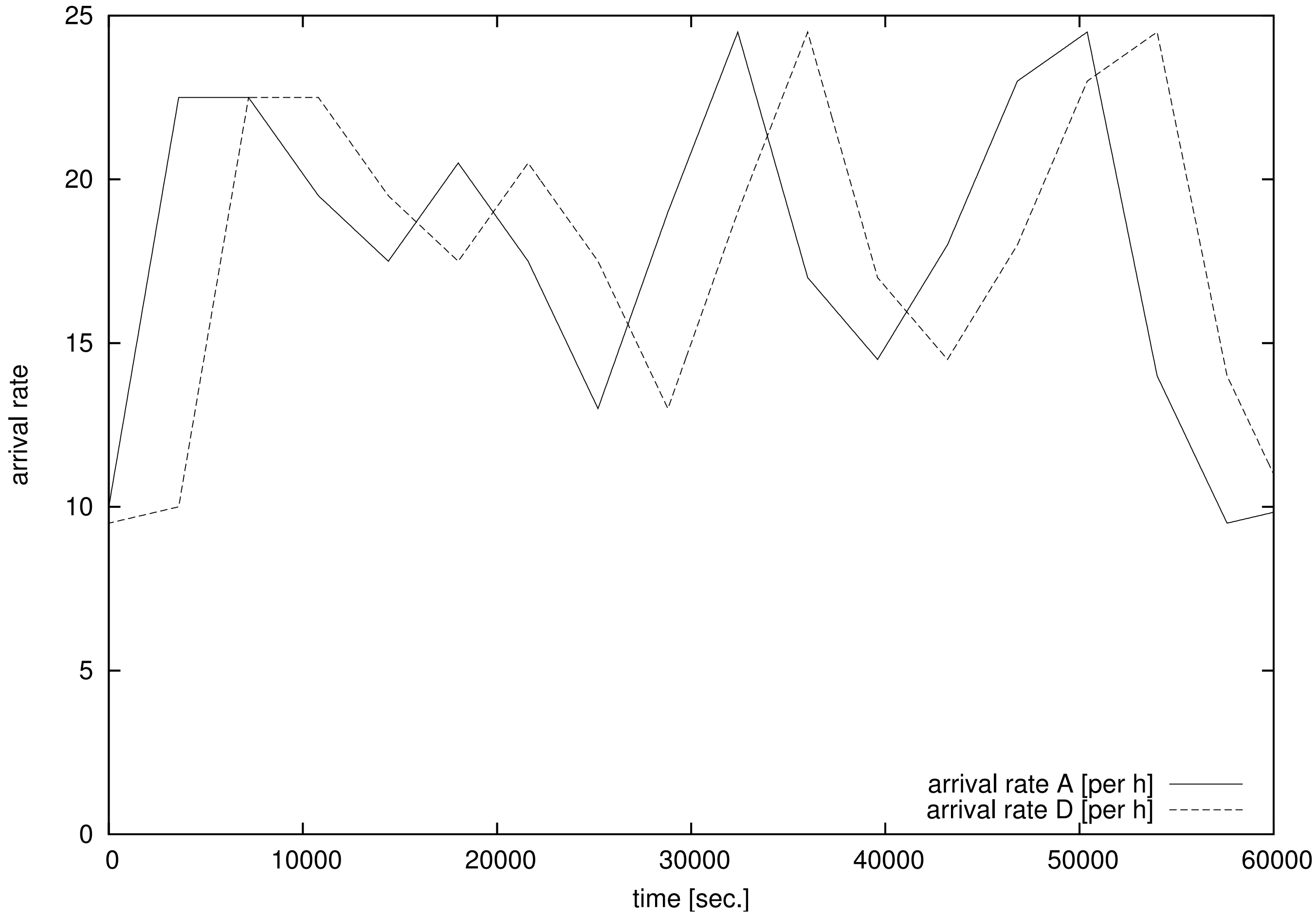
$$\mathbf{E}[L] \quad = \quad \frac{(\lambda_i^{a,MAR} + \lambda_i^{d,MAR})\big(\frac{\lambda_i^{a,MAR}}{\mu^{a2}} + \frac{\lambda_i^{d,MAR}}{\mu^{d2}}\big)}{1 - (\frac{\lambda_i^{a,MAR}}{\mu_i^a} + \frac{\lambda_i^{d,MAR}}{\mu_i^d})}.$$

Numerical Results: Assumptions

- exponentially distributed processing times

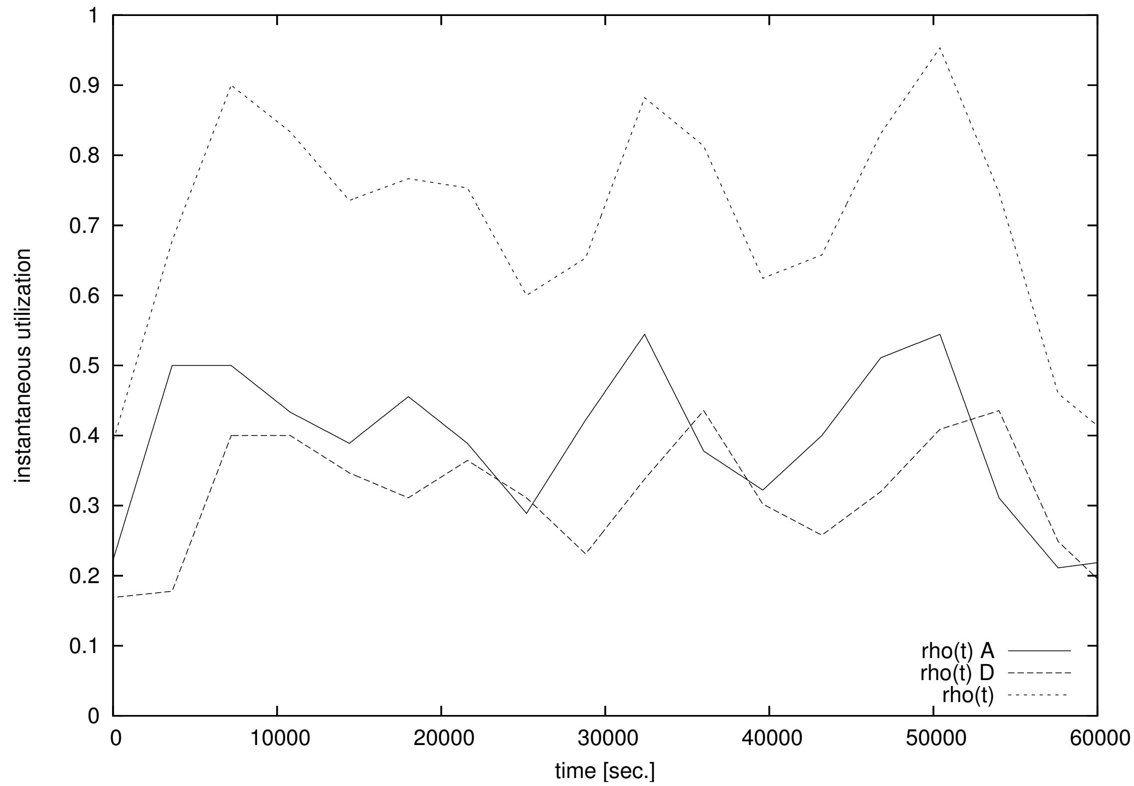
- $\mu_a^{-1} = 80$ and $\mu_d^{-1} = 64$ seconds

- piecewise linear arrival rate functions $\lambda^a(t)$ and $\lambda^d(t)$ (in arrivals per hour)

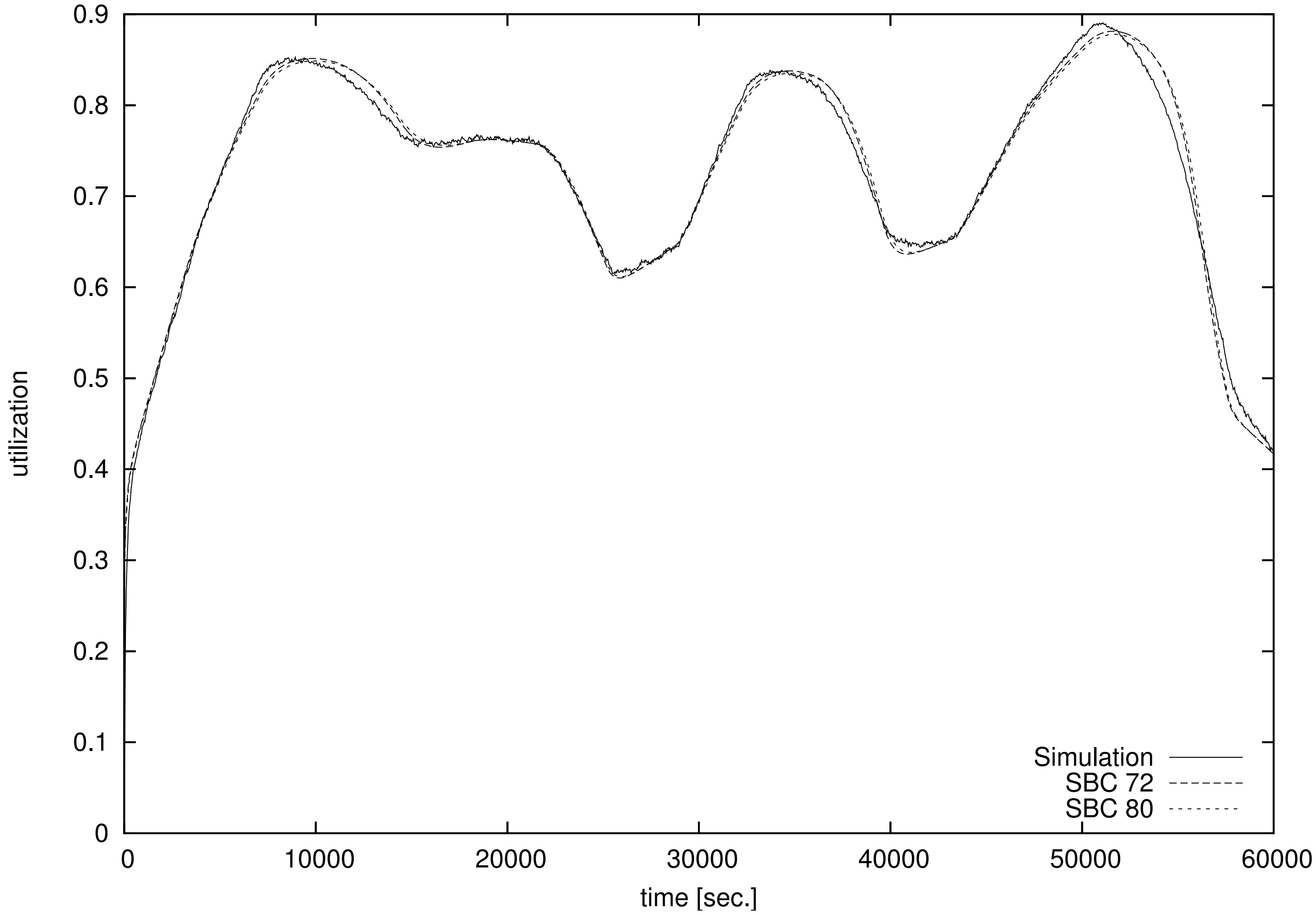


25
20
15
10
5
0
arrival rate
0
10000
20000
30000
40000
50000
60000
time [sec.]
arrival rate A [per h]
arrival rate D [per h]

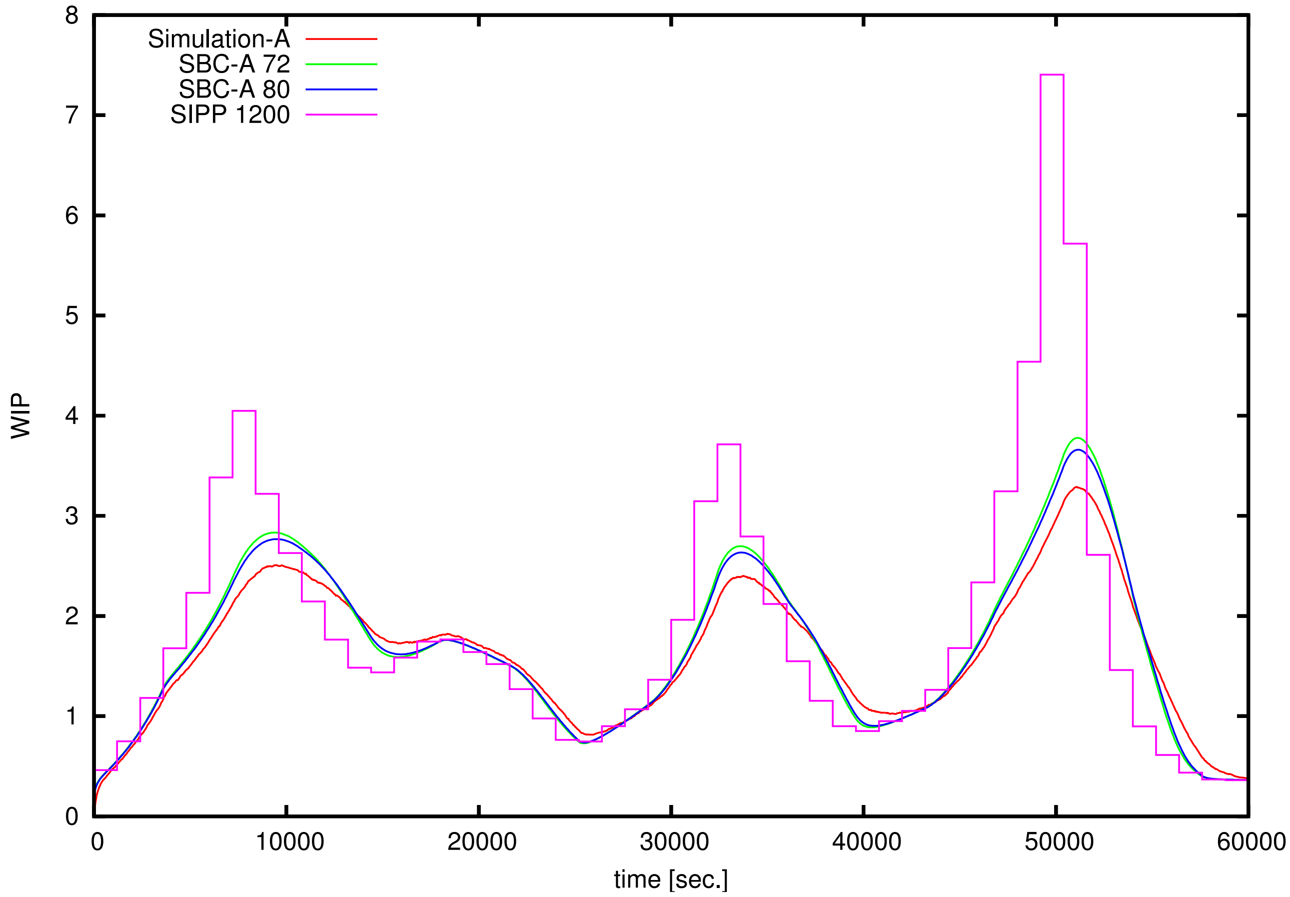
Numerical Results: Instantaneous Traffic Intensity $\rho(t) = \frac{\lambda(t)}{\mu(t)}$



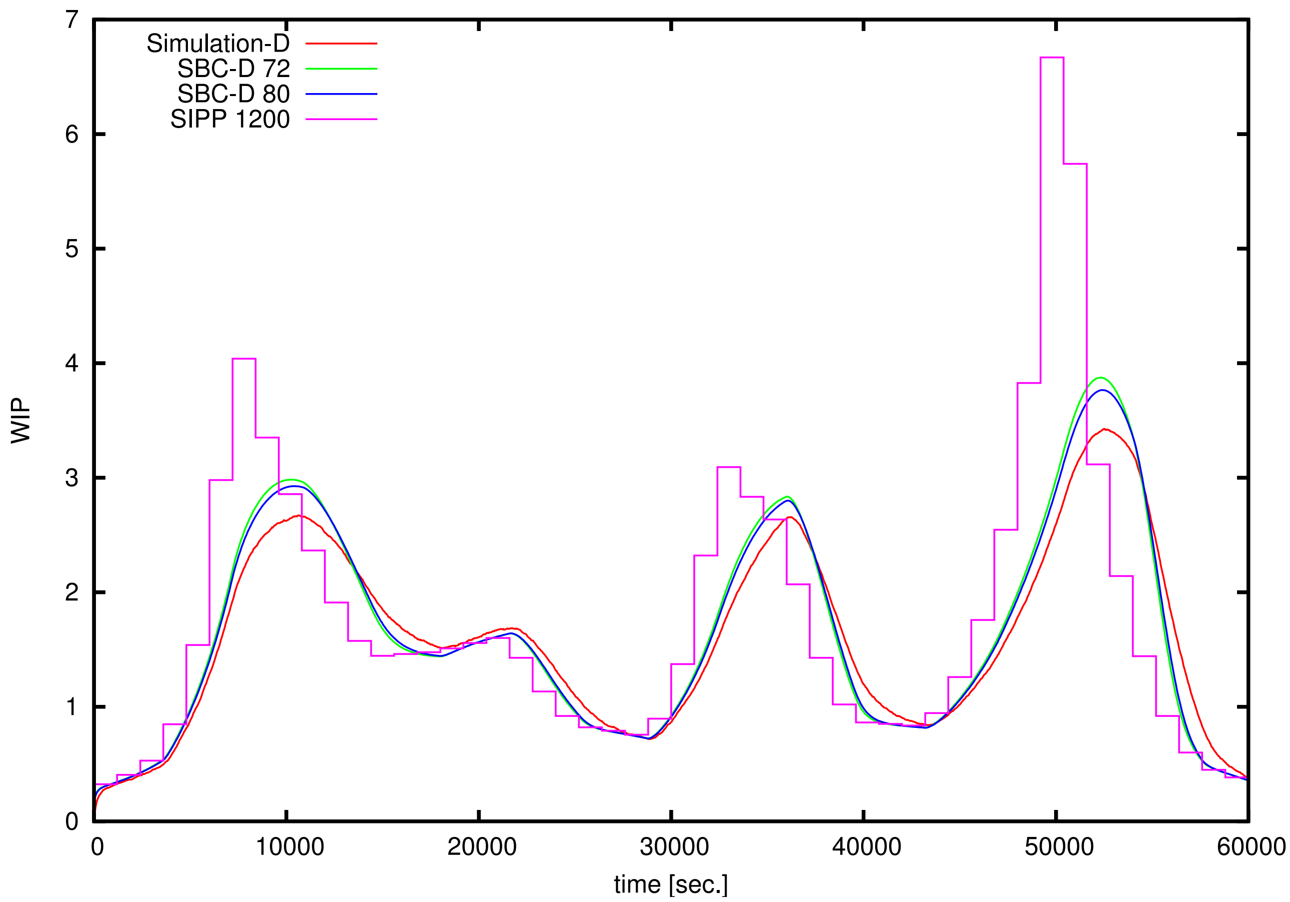
Numerical Results: Utilization



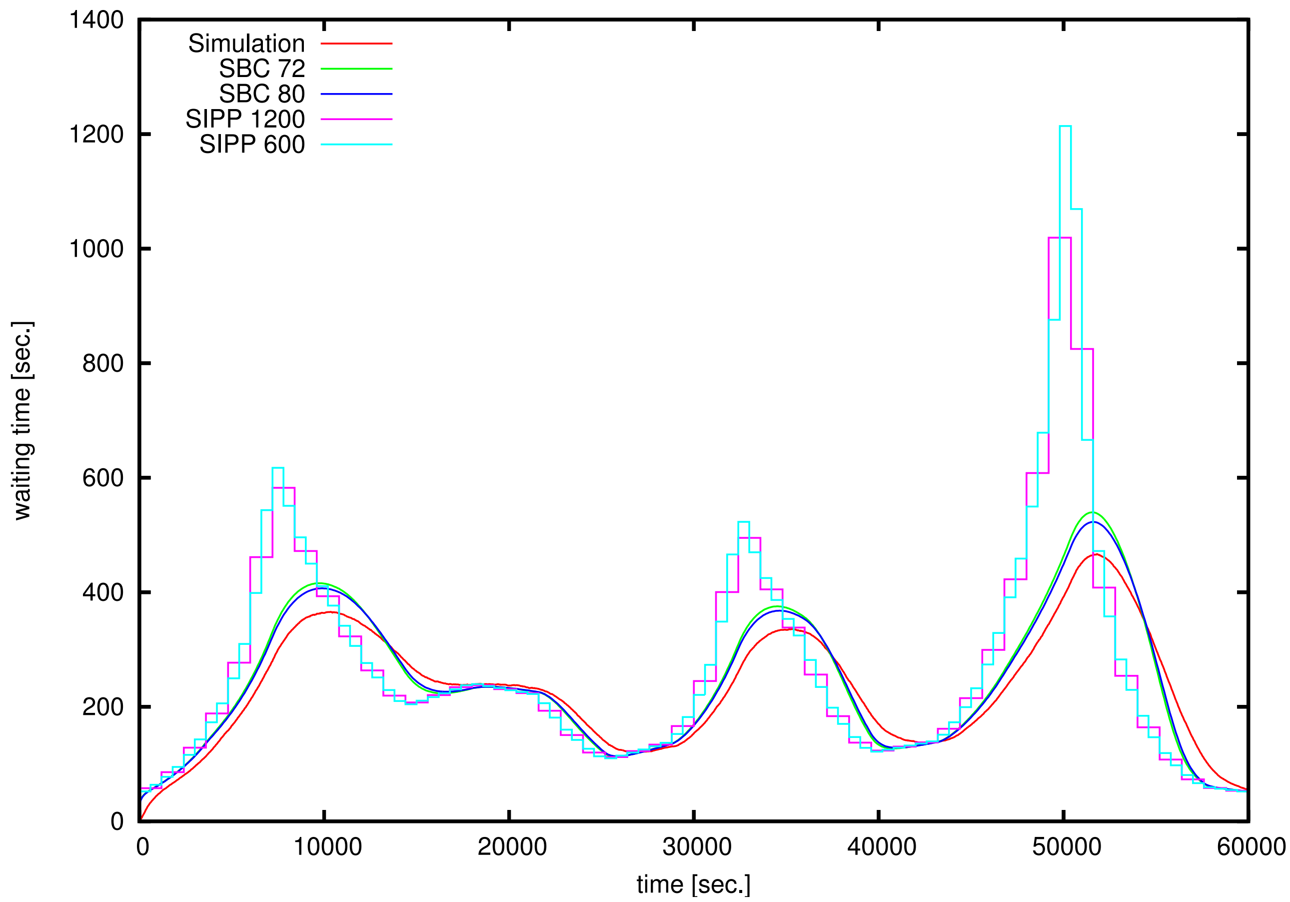
Numerical Results: WIP for Arrivals



Numerical Results: WIP for Departures



Numerical Results: Waiting Time



Conclusion (1)

Possible extensions:

- multiple runways
- different service time distribution
- sequence-dependent separation
- priority service rules

Conclusion (2)

The Stationary Backlog-Carryover (SBC) approach

- is a computationally fast method,
- gives reasonable results for underloaded and partially overloaded systems, and
- is applicable to complex systems.

Thank you for your attention!

raik.stolletz@prod.uni-hannover.de

*Literatur

Grassmann, W. K. (1977). Transient solutions in markovian queueing systems. *Computers & Operations Research 4*(1), 47–53.

Green, L. and P. Kolesar (1991). The pointwise stationary approximation for queues with nonstationary arrivals. *Management Science 37*(1), 84–97.

Green, L., P. Kolesar, and A. Svoronos (1991). Some effects of nonstationarity on multiserver markovian queueing systems. *Operations Research 39*(3), 502–511.

Green, L. V., P. J. Kolesar, and J. Soares (2001). Improving the SIPP approach for staffing service systems that have cyclic demands. *Operations Research 49*(4), 549–564.

Ingolfsson, A., E. Akhmetshina, S. Budge, Y. Li, and X. Wu (2003). A survey and experimental comparison of service level approximation methods for non-stationary M/M/s queueing systems. Technical report, University of Alberta, Edmonton.

Jennings, O. B., A. Mandelbaum, W. A. Massey, and W. Whitt (1996). Server staffing to meet time-varying demand. *Management Science 42*(10), 1383–1394.

Mandelbaum, A. and W. A. Massey (1995). Strong approximations for time-dependent queues. *Mathematics of Operations Research 20*(1), 33–64.

Massey, W. A. and W. Whitt (1997). Uniform acceleration expansions for markov chains with time-varying rates. *Annals of Appl. Prob.*.