

Measuring connectivity in air transport networks: technical description of the available models

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Abstract

Air transport literature has put forward a wide range of models for measuring connectivity in air transport networks. Burghouwt & Redondi (2010) compare and assess the most frequently used connectivity measures. This research note is a technical description of the models used in their paper. It reports mathematical formulations of eight different models to measure hub connectivity and airport accessibility. Even if it does not add to literature, it may be useful for researchers and practitioners, since it provides a general framework to compute the different connectivity measures.

1. Introduction

The widespread use of hub-and-spoke networks has boosted a growing body of literature studying connectivity in air transport system as a separate variable from traditional size-based measures, as number of destinations, flights or offered seats.

This technical note describes the mathematical formulations of eight different models of connectivity as defined in Table 1. Each model can yield measures on both passenger accessibility and hub centrality. The first considers the number and quality of direct and indirect air travel connections available to the consumer at a certain airport. The second measures the number of transfer opportunities available via a specific airport (figure 1).

Model	Short definition	Main references
Hub potential	Incoming * outgoing frequency	Dennis (1998)
'Doganis and Dennis' connectivity	Number of connections. Indirect connections meet conditions of minimum and maximum connecting time and routing factor.	Dennis and Doganis (1989); Dennis (1994a, 1994b)
'Bootsma' connectivity	Number of connections. Indirect connections meet conditions of minimum and maximum connecting time and are classified as 'excellent', 'good' or 'poor'.	Bootsma (1997)
WNX (weighted number of connections)	Number of direct and indirect connections weighted by their quality in terms of transfer and detour time.	Burghouwt and De Wit (2004); Burghouwt (2007)
Netscan connectivity units	Number of direct and indirect connections weighted by their quality in terms of transfer and detour time relative to a theoretical direct flight.	Veldhuis (1997); Burghouwt and Veldhuis (2006); Matsumoto et al. (2008); Veldhuis and Kroes (2002)
Danesi connectivity	Number of direct and indirect connections weighted by their quality in terms of transfer and detour time.	Danesi (2006)
Shortest Path Length centrality	Number of connections lying on O-D shortest paths. The shortest path is the path involving the minimum number of steps from O to D.	Cronrath et al. (2008); Malighetti et al. (2008); Shaw (1993), Shaw and Ivy (1994)
Quickest Path Length centrality	Number of connections lying on O-D quickest paths. The quickest path is the path involving the lower travel time from O to D.	Malighetti et al. (2008); Paleari et al. (2010)
Gross vertex connectivity	Sum of all possible paths with three or fewer flight segments, weighted by a scalar value.	Ivy (1993); Ivy et al. (1995)
Number of connection patterns	Number of statistically significant patterns of incoming and outgoing flights.	Budde et al. (2008)

Table 1. Connectivity measures, definition and studies

The first part of this note identifies the general procedure to compute a measure of hub connectivity and accessibility. The last section of this note reports a short technical profile for each measure.

2. The general procedure

2.1 Hub connectivity models

All eight measures of hub connectivity have the same underlying principles. They can be computed following a two-step procedure. The hub connectivity measure of the intermediate airport i , shown to the left of Figure 6, is computed as follows:

- 1) identify the connections from the generic airport k to the generic airport j passing through airport i that meet some defined conditions that vary from measure to measure. We call those conditions “cut-point” conditions and the resulting connections “viable” connections.
- 2) after indentifying the viable connections, the measure can be obtained by applying the following expression:

$$\text{Hub connectivity measure} = \sum_1^n f(c_{j-i-k})$$

Where n is the number of viable connections and $f(c_{j-i-k})$ is a function of the characteristics of the generic viable connection $j-i-k$ that we call weighting function. It also depends on the specific measure applied.

2.2 Accessibility models

The six local measures of accessibility have the same underlying principles. They can be computed following a two-step procedure. The accessibility measure of an airport i , shown on the right of Figure 1, is computed as follows:

- 1) identify in any airport j , directly linked to airport i , all the connections starting from airport i and going onwards to the generic airport k that meet some defined conditions, varying from measure to measure. Again, we call those conditions “cut-point” conditions and the resulting connections “viable” connections.
- 2) after indentifying the viable connections, the measure can be obtained by applying the following expression:

$$\text{Accessibility measure} = d + \sum_{j=1}^m \sum_1^{n_j} f(c_{i-j-k})$$

The first term d is the direct connectivity, measured as the number of flights from airport i . The second term refers to indirect connectivity, or onward 2-step connectivity, where m is the number of airports with incoming flights from airport i , and n_j is the number of viable connections identified in the intermediate airport j ; $f(c_{i-j-k})$

k) is a function of the characteristics of the generic viable connection j-i-k that we call weighting function. It also depends on the specific measure applied.

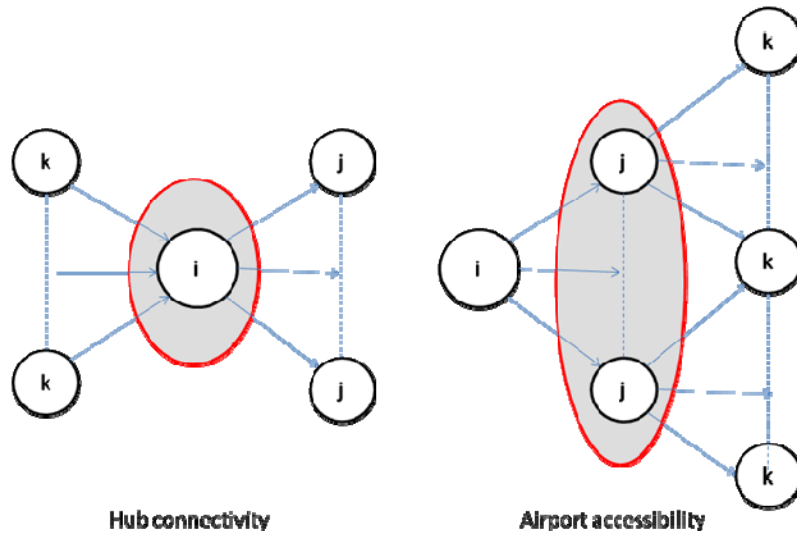


Figure 1. Hub connectivity and airport accessibility measures.

3. Measures profiles

In order to compute all the eight hub connectivity measures and the six local accessibility measures, one only requires to know the cut-point conditions and the particular form of the weighting function that will be reported in the following measures profiles. The remaining two accessibility measures related to SPL and QPL will be considered at the end of this appendix.

3.1 Weighted connectivity

Measure	Weighted connectivity
Main reference	Burghouwt, G. and J. de Wit (2005)
Applications	Hub connectivity and airport accessibility
Cut-point conditions	<ul style="list-style-type: none"> - minimum connecting time (mct) of 60' for all connections - maximum connecting time (MCT) of 180' for EU connections - maximum connecting time (MCT) of 720' for intercontinental connections - maximum routing factor (R) of 1.4 based on flight times
Weighting function for every	$f=WI= \frac{2.4 * TI + RI}{3.4}$; WI: weighted indirect connection;

viable connection	$TI = 1 - \frac{1}{MCT - mct} T$; TI: transfer index; T: connection transfer time; MCT maximum connecting time for the connection; mct: minimum connecting time for the connection; $RI = 1 - (2 * \frac{R}{2} - 2 * \frac{1}{2})$; RI: routing index; R: routing factor; $R = IDT / DTT$ IDT: actual in-flight time; DTT: estimated in-flight time of the direct connection based on the great circle distance	
Software	Microsoft Access	Medium complexity

3.2 Netscan

Measure	Netscan	
Main reference	Veldhuis (1997)	
Applications	Hub connectivity and airport accessibility	
Cut-point conditions	- minimum connecting time (mct) of 60' for all connections	
Weighting function for every viable connection	$f = QUAL = 1 - \frac{PTT - NST}{MAXT - NST}$; QUAL: quality index; NST: non-stop travel time (hours); $PTT = FLY + 3 * TRF$; PTT: Perceived travel time (hours); TRF: Connection transfer time (hours); FLY: Flying time (hours); $MAXT = (3 - 0.075 * NST) * NST$; MAXT: Maximum perceived travel time (hours)	
Software	Microsoft Access	Medium complexity

3.3 Bootsma connectivity

Measure	Bootsma connectivity	
Main reference	Bootsma (1997)	
Applications	Hub connectivity and airport accessibility	
Cut-point conditions	- minimum connecting time (mct) of 60' for all connections - maximum connecting time (MCT) of 180' for EU connections - maximum connecting time (MCT) of 300' for connections from (to) EU to (from) intercontinental airports	

	- maximum connecting time (MCT) of 720' for connections from and to intercontinental airports	
Weighting function for every viable connection	f=1	
Software	Microsoft Access	Low complexity

3.4 Danesi connectivity

Measure	Danesi Connectivity	
Main reference	Danesi (2006)	
Applications	Hub connectivity and airport accessibility	
Cut-point conditions	<ul style="list-style-type: none"> - minimum connecting time (mct) of 60' for all connections - maximum connecting time (MCT) of 120' for EU connections - maximum connecting time (MCT) of 180' for all other connections 	
Weighting function for every viable connection	<p>f=tau*delta; tau: connection time weight; delta: routing factor weight;</p> $\tau = \begin{cases} \text{if } \cdot CT_{EU} < 90' \text{ or } \cdot CT_{INT} < 120' \Rightarrow \tau = 1 \\ \text{otherwise } \cdot \tau = 0.5 \end{cases};$ <p>CT_{EU}=Connecting transfer time for European connections; CT_{INT}= Connecting transfer time for all other connections;</p> $\delta = \begin{cases} \text{if } \cdot RF < 1.2 \Rightarrow \delta = 1 \\ \text{otherwise } \cdot \delta = 0.5 \end{cases};$ <p>RF: routing factor defined as the ratio between the direct distance and the flight's distance;</p>	
Software	Microsoft Access	Medium complexity

3.5 Doganis and Dennis connectivity

Measure	Doganis and Dennis connectivity	
Main reference	Doganis and Dennis (1989)	
Applications	Hub connectivity and airport accessibility	
Cut-point conditions	<ul style="list-style-type: none"> - minimum connecting time (mct) of 60' for all connections - maximum connecting time (MCT) of 90' for all connections 	
Weighting function for every viable connection	f=1	

Software	Microsoft Access	Low complexity
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3.6 Number of connections patterns

Measure	Number of connections patterns	
Main reference	Budde, A., J. de Wit and G. Burghouwt (2008)	
Applications	Hub connectivity and airport accessibility	
Cut-point conditions	<ul style="list-style-type: none"> - minimum connecting time (mct) of 60' for all connections - the connection must be recognized as a statistically significant pattern (see below for more information) 	
Weighting function for every viable connection	f=1	
Software	Matlab	High complexity

Further notes on the number of connections patterns measure

This methodology was originally developed for behavioural research. It was originally devised by the psychologist Magnus Magnusson (2000) for recognising patterns in the occurrence of events.

The algorithm is based on the following principle. If two events occur in succession, (event A followed by B) and do so at least twice within a given timeframe, the program tests the null hypothesis that these events are distributed independently (by chance) and have a constant probability per time unit NB/T (where NB = the number of points of B and T = the observation period in time units). Obviously, in the case of hub schedules, events (departures and arrivals) will rarely be distributed by chance and significance levels will have to be set accordingly high. After setting a significance level, the methodology finds the interval within which event A is followed significantly more often by event B than can be expected by chance. The critical interval research algorithm is analysed in Magnusson (2000) on p.108-109 and the statistical test on p.107. Whenever an event A is followed by event B within a critical interval at least twice within the given timeframe, a pattern (AB) is found. Arrivals and departures can be conceptualised as events. A high quality indirect connection can be conceptualised as a pattern because it consists of two events that occur repeatedly and in close temporal proximity. The inclusion of a flight in a departure/arrival pattern we term pattern participation. An efficiently designed hub schedule will generate a maximum of high quality

indirect connections (patterns) out of a minimum of arrivals and departures (events). A highly connective flight will have a high degree of pattern participation.

3.7 Shortest Path Length - SPL

Measure	SPL – Shortest Path Length	
Main reference	Guimerà et al. (2005)	
Applications	Hub connectivity	
Cut-point conditions	- The connection must lie on the shortest path, in terms of number of steps, from origin to destination	
Weighting function for every viable O-D connection	f=1	
Software	Matlab	High complexity

Further notes on the SPL hub connectivity measure

In order to quantify an airport role as an intermediate step between airports that are not directly connected, graph theory has developed the SPL hub connectivity measure, known as Betweenness centrality (Freeman, 1977).

Guimerà et al. (2005) define the Betweenness of airport *i* as the number of shortest path lengths (SPL) where airport *i* is an intermediate node. Betweenness expresses the centrality of the airport. In many cases, a given pair of airports is connected by several minimal paths with the same number of steps. The Betweenness centrality simply counts all the shortest path lengths that transit through airport *i*, including equivalent alternatives.

3.8 Quickest Path Length - QPL

Measure	QPL – Quickest Path Length	
Main reference	Malighetti et al. (2008)	
Applications	Hub connectivity	
Cut-point conditions	<ul style="list-style-type: none"> - minimum connecting time (mct) of 60' for all connections - maximum routing factor (R) of 1.25 based on distances - the connection must lie on the quickest path, in terms of travel time, from origin to destination 	
Weighting function	f=1	

for every viable O-D connection	$R = [\text{O-D direct distance}] / [\text{in-flight distance}]$	
Software	Matlab	Very high complexity

Further notes on the QPL hub connectivity measure

The problem of the quickest path may be tackled by applying the time-dependent minimum path approach. For more information on these methods, see Miller-Hooks and Patterson (2004). Optimal travel times incorporate both flight time and waiting time at any intermediate airports. The latter may be influenced by several factors, such as the presence of dedicated facilities to manage transfer passengers, airport congestion, and airport size. As said before, in this paper we assume a minimum connecting time of 60 minutes for all airports. This period is acceptable for European connections, but should be lengthened if our analysis is extended to intercontinental flights. We do not exclude any routes on the basis of their connecting times, since the “shortest” path between two airports (in terms of the number of flights required) is always the quickest. If we were to exclude these routes, some of the airports would no longer have a feasible connection.

This analysis also depends on the starting time of each flight. For each pair of airports this model calculates the shortest travel time QPL_{ijt} from airport i to airport j , starting at a specified time t . The day is divided into 96 units of fifteen minutes, so that starting times range from 00:00 to 23:45 (Brussels time). Itineraries ending after midnight are not taken into account. Therefore, for every possible combination of two airports, the model computes the shortest travel times for all flights leaving as early as 00:00 and concluding before midnight of the next day. The minimum travel time for airports i and j is then simply

$$QPL_{ij} = \min(QPL_{ijt}).$$

In order to evaluate hub connectivity, the optimal path from airport i to airport j is defined as the path that: 1) lasts the minimum travel time QPL_{ijt} , and 2) involves the fewest possible steps. For example, if there are two connections from A to B lasting for 5 hours, $A-C-D-B$ and $A-E-B$, only the latter will be defined as optimal.

Shortest and quickest path accessibility models

The network-based models do not express accessibility in just one value. Both the SPL shortest path length and the QPL quickest travel time report a first value indicating how many airports can be reached by departing from a specific airport and a second value indicating how

long is the average path to reach the connected airports. The latter is the average number of steps for the SPL model and the average travel time for the Malighetti et al. model. However, to rank airports based on accessibility those variables must be considered together. To that end, for the shortest path length approach an accessibility index is defined as follows:

$$\text{Accessibility}_{\text{SPL}} = \sum_{j \in N_i} \frac{1}{\text{SPL}_{i,j}}$$

Where N_i represents the set of airports that can be reached from airport i and $\text{SPL}_{i,j}$ is the shortest path length, in terms of number of steps, from airport i to airport j . The index represents the accessibility connection in terms of the equivalent number of one-step connections. For example, if an airport can reach only three other airports with SPL equal to 1, 2 and 2 respectively, the equivalent number of one-step connections is 2 ($1/1+1/2+1/2$) since n -step connections weigh $1/n$ of single step connections.

Analogously, an accessibility index for the Malighetti et al. model can be defined as follows:

$$\text{Accessibility}_{\text{QPL}} = \sum_{j \in N_i} \frac{60}{\text{QPL}_{i,j}}$$

Where again N_i represents the set of airports that can be reached from airport i and $\text{QPL}_{i,j}$ is the quickest travel time, in minutes, from airport i to airport j . The index represents the accessibility connection in terms of the equivalent number of one-hour connections.

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